9.4: PROBLEM DEFINITION

Situation:

A 39 cm block on side weighing 110 N slides on an oil film with thickness of 0.11 mm.

Find:

Terminal velocity of block.

Properties:

Viscosity is 10^{-2} N·s/m^2

PLAN

Apply equilibrium. Then relate shear force (viscous drag force) to viscosity and solve the resulting equation.

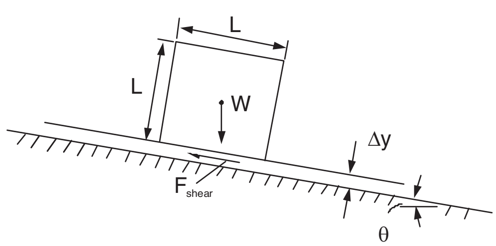
SOLUTION

Force equilibrium

$$\begin{aligned} F_{\text{shear}} &= W \sin\theta \\ \tau &= \frac{F_{\text{shear}}}{A_s} = \frac{W\sin\theta}{L^2} \end{aligned}$$

Shear stress

$$\tau = \mu \frac{dV}{dy} = \mu \times \frac{V}{\Delta y}$$



or

$$V = \frac{\tau \Delta y}{\mu}$$

Then

$$\begin{aligned} V &= \frac{W\sin\theta}{L^2}\frac{\Delta y}{\mu} \\ V &= \left(\frac{110\sin 10^\circ}{(0.39\,\text{m})^2}\right) \times 1.1 \times 10^{-4}/10^{-2} \end{aligned}$$

$$\boxed{V = 1.38 \text{ m/s}}$$

9.8: PROBLEM DEFINITION

Situation:
Uniform, steady flow occurs between two plates.

Find:
(a) Conditions present to cause odd velocity distribution.
(b) Location of minimum shear stress.

SOLUTION

(a) Answer $\Longrightarrow$Pressure distribution causing flow in x-direction but upper plate is moving to the left relative to the lower plate.

(b) Answer $\Longrightarrow$Minimum shear stress occurs where the maximum velocity occurs (where $du/dy = 0$).

9.15: PROBLEM DEFINITION

Situation:

A bearing turns at 200 rad/s inside a 30-mm diameter cylinder 1 cm long. The distance between the shaft and cylinder is 1 mm and filled with SAE 30 oil.

Find:

Torque required to turn bearing.

Properties:

Viscosity from Table A.4 is 0.1 N·s/m^2

SOLUTION

$$\begin{aligned} \tau &= \frac{\mu V}{\delta} \\ T &= \tau A r \end{aligned}$$

where T = torque, A = bearing area = $2\pi r b$

$$\begin{aligned} T &= \tau 2\pi r b r = \tau 2\pi r^2 b \\ &= \frac{\mu V}{\delta}(2\pi r^2 b) \end{aligned}$$

where V=$r\omega$. Then

$$\begin{aligned} T &= \frac{\mu}{\delta}(r\omega)(2\pi r^2 b) \\ &= \frac{\mu}{\delta}(2\pi\omega)r^3 b \\ &= \left(\frac{0.1 \text{ N}\cdot\text{s/m}^2}{0.001 \text{ m}}\right)(2\pi)(200 \text{ rad/s})(0.014 \text{ m})^3(0.01 \text{ m}) \\ &\boxed{T = 3.45\times10^{-3}\,\text{N}\cdot\text{m}} \end{aligned}$$

9.19: PROBLEM DEFINITION

Situation:

Flow occurs between two plates separated by 0.0045 m has a pressure gradient of −3650 Pa/m.

Find:

Maximum fluid velocity in $x-$direction.

Properties:

Viscosity is 0.05 N-s/m^2

PLAN

Use Eq. (9.7a, in 10e) with no change in elevation

SOLUTION

$$\begin{aligned} u_{\max} &= -\left(\frac{B^2}{8\mu}\right)\left(\frac{dp}{ds}\right) \\ &= -\left[\frac{(0.0045 \text{ m})^2}{8 \times 0.05 \text{ N-s/m}^2}\right] \times 3650 \text{ N/m}^3 \end{aligned}$$

$$\boxed{u_{\max} = 0.18 \text{ m/s}}$$

9.26: PROBLEM DEFINITION

Situation:

Couette flow of liquid with temperature and viscosity distribution. The viscosity varies as

$$\mu = \mu_o \exp\left(-0.1\frac{y}{L}\right)$$

Find:

Find shear stress in the form $\tau = C(U\mu_o/L)$

SOLUTION

In a Couette flow the shear stress is constant between plates so

$$\tau = \mu\frac{du}{dy} = \mu_o \exp\left(-0.1\frac{y}{L}\right)\frac{du}{dy} = const$$

Separating variables

$$\exp\left(0.1\frac{y}{L}\right)d\left(\frac{y}{L}\right) = \frac{\mu_o}{\tau L}du$$

Integrating

$$\begin{aligned}
\int_0^1 \exp\left(0.1\frac{y}{L}\right)d\left(\frac{y}{L}\right) &= \frac{\mu_o}{\tau L}\int_0^U du \\
10\left[\exp\left(0.1\right) - 1\right] &= \frac{\mu_o U}{\tau L} \\
1.052 &= \frac{\mu_o U}{\tau L}
\end{aligned}$$

$$\boxed{\tau = 0.951\frac{\mu_o U}{L}}$$

9.42: PROBLEM DEFINITION

Situation:

Water at 20 °C flows over a flat plate 1.5 m long and 1.0 m wide at 15 cm/s.

Find:

(a) Resistance of plate.

(b) Boundary layer thickness at trailing edge.

Properties:

Table A.5 (water at 20 °C): $\rho = 998\,\text{kg/m}^3$.

$\mu = 1.00 \times 10^{-3}\,\text{N}\cdot\text{s/m}^2$, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/$

SOLUTION

Reynolds number

$$\begin{aligned} \text{Re}_L &= \frac{U_0 L}{\nu} \\ &= \frac{0.15 \text{ m/s} \times 1.5 \text{ m}}{10^{-6} \text{ m}^2/\text{s}} \\ &= 2.25 \times 10^5 \end{aligned}$$

$Re_L \leq 500{,}000$; therefore, laminar boundary layer

Boundary layer thickness

$$\begin{aligned} \delta &= \frac{5x}{\text{Re}_x^{1/2}} \\ &= \frac{5 \times 1.5 \text{ m}}{(2.25 \times 10^5)^{1/2}} = 1.58 \times 10^{-2} \text{ m} \\ & \boxed{\delta = 15.8 \text{ mm}} \end{aligned}$$

Average shear stress coefficient

$$\begin{aligned} C_f &= \frac{1.33}{\text{Re}_L^{1/2}} \\ &= \frac{1.33}{(2.25 \times 10^5)^{1/2}} \\ &= 0.00280 \end{aligned}$$

Surface resistance (drag force) - 2 surfaces (top and bottom)

$$\begin{aligned} F_s &= C_f A \frac{\rho U_0^2}{2} \\ &= 0.00280 \times (1.0 \text{ m} \times 1.5 \text{ m} \times 2) \times \frac{998\,\text{kg/m}^2 \times (0.15 \text{ m/s})^2}{2} \\ & \boxed{F_s = 0.0943 \text{ N}} \end{aligned}$$

9.51: PROBLEM DEFINITION

Situation:

Flow over a flat plate with linear velocity profile at trailing edge.

Find:

Skin friction drag on top per unit width stress on plate at downstream end.

Properties:

$\mu = 1.8 \times 10^{-5}$ N·s/m^2

PLAN

Relate velocity profile and shear stress at trailing edge.

SOLUTION

Local shear stress

$$\begin{aligned}\tau_0 &= \mu \frac{dV}{dy} = \mu \frac{V}{\Delta y} \\ &= \frac{1.8 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2 \times 40\ \text{m/s}}{3 \times 10^{-3}\ \text{m}} \\ & \boxed{\tau_0 = 0.24\ \text{N/m}^2}\end{aligned}$$